ATOM
ANTENNA TOOLBOX FOR MATLAB

MoM-Based Matrix Operators in AToM

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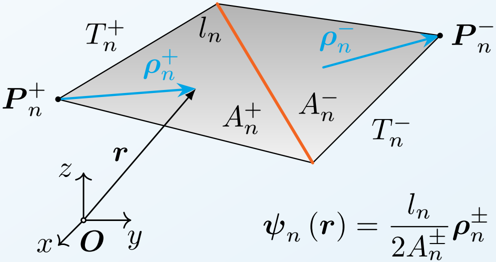
Outline



$$\boldsymbol{\psi}_n\left(\boldsymbol{r}\right) = \frac{l_n}{2A_n^{\pm}}\boldsymbol{\rho}_n^{\pm}$$

Definition of RWG function.

Basic Equations

Quadratic form of linear operator $\mathcal{L}$ reads

$$p = \frac{1}{2} \int\limits_{\Omega} \boldsymbol{J}^* \cdot \mathcal{L}\left\{\boldsymbol{J}\right\} \mathrm{d}\boldsymbol{r}, \quad \boldsymbol{J} = \boldsymbol{J}\left(\boldsymbol{r}\right), \boldsymbol{r} \in \Omega. \tag{1}$$

Complex power is defined as

$$P_{\mathrm{rad}} + 2\mathrm{j}\omega\left(W_{\mathrm{m}} - W_{\mathrm{e}}\right) = -\frac{1}{2} \int\limits_{V} \boldsymbol{J}^* \cdot \boldsymbol{E} \, \mathrm{d}V \tag{2}$$

Electric Field Integral Equation (EFIE) reads[1]

$$\boldsymbol{E}\left(\boldsymbol{r}\right) = -\mathrm{j}Z_0 k \int\limits_{\Omega} \left(\mathbf{1} + \frac{\nabla\nabla\cdot}{k^2}\right) \boldsymbol{J}\left(\boldsymbol{r}'\right) \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R} \, \mathrm{d}S, \tag{3}$$

with $\mathbf{1}$ being the identity matrix, $\mathbf{1} = \left[\delta_{pq}\right]$.

[1] J. L. Volakis and K. Sertel, *Integral equation methods for electromagnetics*. Scitech Publishing Inc., 2012.

Usage of (finite) discretization of the source region Ω leads to

$$\boldsymbol{J}(\boldsymbol{r}) \approx \sum_{n=1}^{N} I_n \boldsymbol{\psi}_n(\boldsymbol{r}), \tag{4}$$

where $\mathbf{I} = [I_n] \in \mathbb{C}^{N\times 1}$ are expansion coefficients, $\boldsymbol{\psi}_n$ are the basis functions.

Formula (4) transforms (1) into algebraic relation

$$p \approx \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{L}\mathbf{I} \quad \text{with} \quad L_{mn} \equiv \int_{\Omega} \boldsymbol{\psi}_m \cdot \mathcal{L}\{\boldsymbol{\psi}_n\}\,\mathrm{d}\boldsymbol{r}. \tag{5}$$

Complex power (2) is approximated as

$$P_{\mathrm{rad}} + 2\mathrm{j}\omega\left(W_{\mathrm{m}} - W_{\mathrm{e}}\right) \approx \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{Z}\mathbf{I}, \tag{6}$$

where $\mathbf{Z} \in \mathbb{C}^{N\times N}$ is the impedance matrix defined as follows.

The impedance matrix is constructed (from normalized data) as

$$\mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X} = \mathrm{j} Z_0 a^2 \left(ka \left(\mathbf{Z}^{M,k} + \mathbf{Z}^{M,0} \right) - \frac{1}{ka} \left(\mathbf{Z}^{E,k} + \mathbf{Z}^{E,0} \right) \right), \tag{7}$$

where individual elements of the defining matrices are

$$Z_{mn}^{M,k} = \frac{1}{a^3} \int\limits_{\Omega} \int\limits_{\Omega} \boldsymbol{\psi}_m \left(\boldsymbol{r} \right) \cdot \boldsymbol{\psi}_n \left(\boldsymbol{r}' \right) \frac{\mathrm{e}^{-\mathrm{j}kR} - 1}{4\pi R} \, \mathrm{d}S \, \mathrm{d}S', \tag{8}$$

$$Z_{mn}^{M,0} = \frac{1}{a^3} \int\limits_{\Omega} \int\limits_{\Omega} \boldsymbol{\psi}_m \left(\boldsymbol{r} \right) \cdot \boldsymbol{\psi}_n \left(\boldsymbol{r}' \right) \frac{1}{4\pi R} \, \mathrm{d}S \, \mathrm{d}S', \tag{9}$$

$$Z_{mn}^{E,k} = \frac{1}{a} \int\limits_{\Omega} \int\limits_{\Omega} \nabla \cdot \boldsymbol{\psi}_m \left(\boldsymbol{r} \right) \nabla' \cdot \boldsymbol{\psi}_n \left(\boldsymbol{r}' \right) \frac{\mathrm{e}^{-\mathrm{j}kR} - 1}{4\pi R} \, \mathrm{d}S \, \mathrm{d}S', \tag{10}$$

$$Z_{mn}^{E,0} = \frac{1}{a} \int\limits_{\Omega} \int\limits_{\Omega} \nabla \cdot \boldsymbol{\psi}_m \left(\boldsymbol{r} \right) \nabla' \cdot \boldsymbol{\psi}_n \left(\boldsymbol{r}' \right) \frac{1}{4\pi R} \, \mathrm{d}S \, \mathrm{d}S'. \tag{11}$$

Normalization of characteristic modes[2] to unitary radiated power reads

$$\frac{1}{2}\int\limits_{\Omega} \boldsymbol{J}_p^* \mathcal{Z}\left\{\boldsymbol{J}_q\right\} \mathrm{d}\boldsymbol{r} = \left(1 + \mathrm{j}\lambda_p\right)\delta_{pq}. \tag{12}$$

The same normalization based on impedance matrix is[3]

$$\frac{1}{2}\mathbf{I}_p^{\mathrm{H}}\mathbf{Z}\mathbf{I}_q = \left(1 + \mathrm{j}\lambda_p\right)\delta_{pq}. \tag{13}$$

Physical units of impedance matrix $\mathbf{Z}$ are $[\Omega\,\mathrm{m}^2]$.

Matrices $\mathbf{Z}^{M,k}$, $\mathbf{Z}^{M,0}$, $\mathbf{Z}^{E,k}$, $\mathbf{Z}^{E,0}$ are made dimensionless.

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[2] M. Capek, P. Hazdra, M. Masek, *et al.*, "Analytical representation of characteristic modes decomposition," *IEEE Trans. Antennas Propag.*, vol. 65, no. 2, pp. 713–720, 2017. DOI: 10.1109/TAP.2016.2632725.

[3] Complex and Hermitian conjugate can be removed in case as the characteristic modes for PEC bodies are by definition real-valued.

The analytical differentiation of $\mathbf{Z}$ with respect to ω (normalized by ω) is

$$\omega \frac{\partial \mathbf{Z}}{\partial \omega} = \mathrm{j} Z_0 a^2 \left(ka \left(\mathbf{Z}^{M,k} + \mathbf{Z}^{M,0} - \mathrm{j} ka \mathbf{T}^{M} \right) + \frac{1}{ka} \left(\mathbf{Z}^{E,k} + \mathbf{Z}^{E,0} + \mathrm{j} ka \mathbf{T}^{E} \right) \right), \tag{14}$$

where individual elements of the defining matrices are

$$T_{mn}^{M} = \frac{1}{a^4} \int\limits_{\Omega} \int\limits_{\Omega} \boldsymbol{\psi}_m (\boldsymbol{r}) \cdot \boldsymbol{\psi}_n (\boldsymbol{r}') \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi} \,\mathrm{d}S \,\mathrm{d}S', \tag{15}$$

$$T_{mn}^{E} = \frac{1}{a^2} \int\limits_{\Omega} \int\limits_{\Omega} \nabla \cdot \boldsymbol{\psi}_m (\boldsymbol{r}) \, \nabla' \cdot \boldsymbol{\psi}_n (\boldsymbol{r}') \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi} \,\mathrm{d}S \,\mathrm{d}S', \tag{16}$$

and

$$R = |\boldsymbol{r} - \boldsymbol{r}'| , \quad a = \max_{\boldsymbol{r}, \boldsymbol{r}' \in \Omega} \left\{ \frac{R}{2} \right\}, \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{17}$$

The stored energy matrix[4] for $ka < 1$ is defined as

$$\mathbf{W} = \operatorname{Im}\left\{\omega \frac{\partial \mathbf{Z}}{\partial \omega}\right\}. \tag{18}$$

and the stored electric and magnetic energies are

$$\mathbf{X}_\mathrm{m} = \frac{1}{2}\left(\mathbf{W} + \mathbf{X}\right), \tag{19}$$

$$\mathbf{X}_\mathrm{e} = \frac{1}{2}\left(\mathbf{W} - \mathbf{X}\right). \tag{20}$$

Therefore, the following identities holds

$$\mathbf{X} = \mathbf{X}_\mathrm{m} - \mathbf{X}_\mathrm{e}, \tag{21}$$

$$\mathbf{W} = \mathbf{X}_\mathrm{m} + \mathbf{X}_\mathrm{e}. \tag{22}$$

[4] R. F. Harrington and J. R. Mautz, “Control of radar scattering by reactive loading,” *IEEE Trans. Antennas Propag.*, vol. 20, no. 4, pp. 446–454, 1972. DOI: 10.1109/TAP.1972.1140234.

Basis-Function Overlap Matrix and Lossy Matrix

The basis-function overlap matrix $\boldsymbol{\Psi} = [\Psi_{mn}]$ is defined as[5] sparse matrix

$$\Psi_{mn} = \int_{\Omega} \boldsymbol{\psi}_m(\boldsymbol{r}) \cdot \boldsymbol{\psi}_n(\boldsymbol{r}) \, \mathrm{d}S. \tag{23}$$

Properties:

- all eigenvalues are strictly positive (non-negative),
- it is a Gram matrix,
- it is sparse; sparsity for a sphere of T triangles with $N = 3T/2$ RWG basis functions is equal[6] to $1 - 5/N$.

[5] L. Jelinek and M. Capek, "Optimal currents on arbitrarily shaped surfaces," *IEEE Trans. Antennas Propag.*, vol. 65, no. 1, pp. 329–341, 2017. DOI: 10.1109/TAP.2016.2624735.

[6] Topologically minimal S^2 sphere is a 3-simplex (tetrahedron), then $N = 6$ and each basis function $\boldsymbol{\psi}_n$ sees all the others except of one. All opened and simple structures have higher sparsity.

The dissipation (ohmic) losses matrix $\mathbf{R}_\Omega = [R_{\Omega,mn}] \in \mathbb{R}^{N \times N}$ as

$$R_{\Omega,mn} = \int\limits_{\Omega} R_{\mathrm{s}}(\boldsymbol{r})\, \boldsymbol{\psi}_m(\boldsymbol{r}) \cdot \boldsymbol{\psi}_n(\boldsymbol{r}) \, \mathrm{d}S, \tag{24}$$

with $R_{\mathrm{s}}(\boldsymbol{r}) \geq 0$ being spatially dependent surface resistivity. For $R_{\mathrm{s}}(\boldsymbol{r}) = R_{\mathrm{s}}$

$$\mathbf{R}_\Omega = R_{\mathrm{s}} \boldsymbol{\Psi} \tag{25}$$

and inheriting all properties of matrix $\boldsymbol{\Psi}$.

The topological adjacency matrix $\mathbf{A} = [A_{mn}]$ can then be defined as

$$A_{mn} = \begin{cases} 1 & \Leftrightarrow \quad \Psi_{mn} \neq 0, \\ 0 & \Leftrightarrow \quad \Psi_{mn} = 0. \end{cases} \tag{26}$$

The dissipation loss matrix $\mathbf{R}_\Omega$ and matrix of lumped elements $\mathbf{Z}_\mathrm{L} = [Z_{\mathrm{L},mn}]$ can be added to the impedance matrix as

$$\widehat{\mathbf{Z}} = \widehat{\mathbf{R}} + \mathrm{j}\widehat{\mathbf{X}} = \mathbf{R}_\Omega + \mathbf{Z}_\mathrm{L} + \mathbf{Z}. \tag{27}$$

The lumped element matrix elements read

$$Z_{\mathrm{L},nn} = l_n^2 \left(R_n + \mathrm{j} \left(\omega L_n - \frac{1}{\omega C_n} \right) \right). \tag{28}$$

The far-field is defined as

$$F\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}\right) = \lim_{r \to \infty} \left\{ r \mathrm{e}^{\mathrm{j}kr} \hat{\boldsymbol{e}} \cdot \boldsymbol{E}\left(\boldsymbol{r}\right) \right\}. \tag{29}$$

Explicitly,

$$F\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}\right) = -\frac{\mathrm{j} Z_0 k}{4\pi} \int\limits_{\Omega} \hat{\boldsymbol{e}} \cdot \boldsymbol{J}\left(\boldsymbol{r}'\right) \mathrm{e}^{\mathrm{j}k \hat{\boldsymbol{r}} \cdot \boldsymbol{r}'} \, \mathrm{d}S, \tag{30}$$

and

$$U\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}\right) = \frac{1}{2Z_0} \left|F\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}\right)\right|^2, \quad U\left(\hat{\boldsymbol{r}}\right) = U\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\varphi}}\right) + U\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\vartheta}}\right), \tag{31}$$

$$P_{\mathrm{rad}} = \int\limits_{S^2} U\left(\hat{\boldsymbol{r}}\right) \, \mathrm{d}S, \quad D\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}\right) = \frac{4\pi U\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}\right)}{P_{\mathrm{rad}}}. \tag{32}$$

Radiation intensity matrix $\mathbf{U} = [U_{mn}]$ for direction $\hat{\boldsymbol{r}}$ and polarization $\hat{\boldsymbol{e}}$ is defined[7] as

$$U_{mn}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}) = \frac{Z_0 k^2}{32\pi^2} \int_\Omega \int_\Omega (\hat{\boldsymbol{e}} \cdot \boldsymbol{\psi}_m)(\hat{\boldsymbol{e}} \cdot \boldsymbol{\psi}_n) \, \mathrm{e}^{-\mathrm{j}k\hat{\boldsymbol{r}}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \, \mathrm{d}S \, \mathrm{d}S', \tag{33}$$

and it is a rank-one matrix as the far-field vector $\mathbf{F} = [F_n]$ defined as

$$F_n(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}) = \frac{-\mathrm{j}Z_0 k}{4\pi} \int_\Omega \hat{\boldsymbol{e}} \cdot \boldsymbol{\psi}_n \mathrm{e}^{\mathrm{j}k\hat{\boldsymbol{r}}\cdot\boldsymbol{r}} \, \mathrm{d}S \tag{34}$$

can be used

$$\mathbf{U}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}) = \frac{1}{2Z_0} \mathbf{F}^{\mathrm{H}}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}) \, \mathbf{F}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}}) \, . \tag{35}$$

Total radiation intensity can be calculated as

$$\mathbf{U}(\hat{\boldsymbol{r}}) = \mathbf{U}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\varphi}}) + \mathbf{U}\left(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\vartheta}}\right). \tag{36}$$

[7] L. Jelinek and M. Capek, "Optimal currents on arbitrarily shaped surfaces," *IEEE Trans. Antennas Propag.*, vol. 65, no. 1, pp. 329–341, 2017. DOI: 10.1109/TAP.2016.2624735.

Spherical waves can be projected onto basis functions as[8]

$$S_{\alpha n} = k\sqrt{Z_0} \int\limits_{\Omega} \mathbf{u}_{\alpha}^{(1)} \left(k\boldsymbol{r}\right) \cdot \boldsymbol{\psi}_n \left(\boldsymbol{r}\right) \mathrm{d}S, \tag{37}$$

α is multi-index σ, m, l, and $\mathbf{u}_{\alpha}^{(1)} \left(k\boldsymbol{r}\right)$ are the spherical modes[9].

The relationship between resistance part of impedance matrix and spherical waves matrix

$$\mathbf{R} = \mathbf{S}^{\mathrm{T}}\mathbf{S}. \tag{38}$$

[8] D. Tayli, M. Capek, L. Akrou, *et al.*, "Accurate and efficient evaluation of characteristic modes," , 2017, submitted, arXiv:1709.09976. [Online]. Available: `https://arxiv.org/abs/1709.09976`.

[9] G. Kristensson, *Scattering of electromagnetic waves by obstacles*. Edison, NJ: SciTech Publishing, an imprint of the IET, 2016.

Properties of projection matrix S

Matrix $\mathbf{S} \in \mathbb{R}^{N_\alpha \times N}$ is a projection matrix between RWG basis and spherical waves basis. It is low-rank matrix in α, suitable for compression techniques.

For Ω being a spherical shell, the rows of matrix $\mathbf{S}$ contains directly the expansion coefficients of corresponding spherical waves, $\mathbf{I}_\alpha$, since

$$\mathbf{I}_{\alpha p}^{\mathrm{H}} \mathbf{S}^{\mathrm{T}} \mathbf{S} \mathbf{I}_{\alpha q} = K_\alpha \delta_{pq}, \tag{39}$$

where K_α is arbitrary constant depending on scaling.
Real part of the impedance matrix $\mathbf{Z}$ represented in basis of spherical waves is

$$\mathbf{R}_\alpha = \mathbf{S}\mathbf{S}^{\mathrm{T}}. \tag{40}$$

The radiated far-field can be expressed as

$$\boldsymbol{F}(\hat{\boldsymbol{r}}) = \frac{1}{k} \sum_{\alpha} \mathrm{j}^{l-\tau+2} f_\alpha \mathbf{Y}_\alpha(\hat{\boldsymbol{r}}), \tag{41}$$

where $\mathbf{Y}_\alpha(\hat{\boldsymbol{r}})$ are the real-valued spherical vector harmonics and

$$[f_\alpha] = \mathbf{SI}, \quad f_\alpha \in \mathbb{R}^{1\times 1}. \tag{42}$$

Then we get a series of equalities

$$P_{\mathrm{rad}} \approx \frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{RI} = \frac{1}{2}|\mathbf{SI}|^2 = \frac{1}{2}\sum_{\alpha}|f_\alpha|^2 = \mathbf{I}^{\mathrm{H}} \int\limits_{S^2} \Big(\mathbf{U}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\varphi}}) + \mathbf{U}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\vartheta}})\Big)\, \mathrm{d}S\, \mathbf{I}, \tag{43}$$

with S^2 being unit sphere.

Electric and magnetic fields are approximated as

$$\boldsymbol{N}_{\mathrm{e},n}(\boldsymbol{r}) = -\mathrm{j}Z_0 k \int\limits_{\Omega} \left(\left(\hat{\boldsymbol{r}} \times \boldsymbol{\psi}_n \right) \times \hat{\boldsymbol{r}} + \left(3\hat{\boldsymbol{r}} \left(\hat{\boldsymbol{r}} \cdot \boldsymbol{\psi}_n \right) - \boldsymbol{\psi}_n \right) \left(\frac{1}{(kR)^2} + \frac{\mathrm{j}}{kR} \right) \right) \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R} \, \mathrm{d}S, \tag{44}$$

$$\boldsymbol{N}_{\mathrm{m},n}(\boldsymbol{r}) = -\mathrm{j}k \int\limits_{\Omega} \left(\hat{\boldsymbol{r}} \times \boldsymbol{\psi}_n \right) \left(1 + \frac{1}{\mathrm{j}kR} \right) \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R} \, \mathrm{d}S. \tag{45}$$

with $\mathbf{N}_{\mathrm{e}} = \left[\boldsymbol{N}_{\mathrm{e},n} \right], \mathbf{N}_{\mathrm{m}} = \left[\boldsymbol{N}_{\mathrm{m},n} \right] \in \mathbb{C}^{N \times 3}$.

Notice then that

$$\boldsymbol{E}(\boldsymbol{r}) \approx \mathbf{N}_{\mathrm{e}}(\boldsymbol{r}) \mathbf{I}, \tag{46}$$

$$\boldsymbol{H}(\boldsymbol{r}) \approx \mathbf{N}_{\mathrm{m}}(\boldsymbol{r}) \mathbf{I}. \tag{47}$$

Electric dipole moment matrix $\mathbf{p} = [\boldsymbol{p}_n] \in \mathbb{R}^{N\times 3}$ is defined as

$$\boldsymbol{p}_n = \frac{\mathrm{j}}{\omega} \int\limits_{\Omega} \boldsymbol{r} \nabla \cdot \boldsymbol{\psi}_n \, \mathrm{d}S = \frac{1}{\mathrm{j}\omega} \int\limits_{\Omega} \boldsymbol{\psi}_n \, \mathrm{d}S. \tag{48}$$

Magnetic dipole moment matrix $\mathbf{m} = [\boldsymbol{m}_n] \in \mathbb{R}^{N\times 3}$ is defined as

$$\boldsymbol{m}_n = \frac{1}{2} \int\limits_{\Omega} \boldsymbol{r} \times \boldsymbol{\psi}_n \, \mathrm{d}S \tag{49}$$

Squared magnitude of the electric and magnetic dipole moments can be calculated as

$$|\boldsymbol{p}|^2 = \mathbf{I}^{\mathrm{H}} \mathbf{P} \mathbf{I}, \tag{50}$$

$$|\boldsymbol{m}|^2 = \mathbf{I}^{\mathrm{H}} \mathbf{M} \mathbf{I}, \tag{51}$$

where matrices $\mathbf{P} \in \mathbb{R}^{N\times N}$ and $\mathbf{M} \in \mathbb{R}^{N\times N}$ are

$$\mathbf{P} = \mathbf{p}\mathbf{p}^{\mathrm{H}}, \tag{52}$$

$$\mathbf{M} = \mathbf{m}\mathbf{m}^{\mathrm{H}}. \tag{53}$$

Useful relations[10] are

$$\nabla \cdot \boldsymbol{\psi}_n \left(\boldsymbol{r}\right) = \pm \frac{l_n}{A_n^{\pm}}, \tag{54}$$

$$\int\limits_{\Omega} \boldsymbol{\psi}_n \left(\boldsymbol{r}\right) \, \mathrm{d}S = l_n \left(\boldsymbol{r}_n^{c+} - \boldsymbol{r}_n^{c-}\right), \tag{55}$$

$$\int\limits_{\Omega} \boldsymbol{r} \times \boldsymbol{\psi}_n \, \mathrm{d}S = \frac{l_n}{2} \left(\mathbf{P}_n^{+} \times \boldsymbol{r}_n^{c+} - \mathbf{P}_n^{-} \times \boldsymbol{r}_n^{c-}\right), \tag{56}$$

$$\int\limits_{\Omega} \nabla \cdot \boldsymbol{\psi}_m \left(\boldsymbol{r}\right) \, \mathrm{d}S = \frac{l_n}{A_n^{+}} \int\limits_{A_n^{+}} \mathrm{d}S - \frac{l_n}{A_n^{-}} \int\limits_{A_n^{-}} \mathrm{d}S = 0, \tag{57}$$

where $\Omega = A_n^{+} \cup A_n^{-}$.

[10] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, vol. 30, no. 3, pp. 409–418, 1982. DOI: 10.1109/TAP.1982.1142818.

For $\boldsymbol{r} \equiv [x \; y \; z]$ the transformation from/to simplex coordinates (α, β, γ) reads

$$\boldsymbol{r} = \begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix} \begin{bmatrix} \mathbf{P}_A \\ \mathbf{P}_B \\ \mathbf{P}^{\pm} \end{bmatrix}. \tag{58}$$

Notice that $\gamma = 1 - \alpha - \beta$, and

$$\int\limits_T f(\boldsymbol{r}) \, \mathrm{d}S = 2A \int\limits_0^1 \int\limits_0^{1-\alpha} f\left(\alpha \mathbf{P}_A + \beta \mathbf{P}_B + (1 - \alpha - \beta) \mathbf{P}^{\pm}\right) \mathrm{d}\beta \, \mathrm{d}\alpha. \tag{59}$$

Discretization grid and RWG basis functions:

Listing 1: atom_code1_mesh.m

```
% Frequency
f = atom.selectedProject.physics.getFrequencyListValues;
% or manually specified by user:
f = 1e9; % 1 GHz

% (It is expected that MoM requests are correctly set up.)
Res = atom.selectedProject.solver.MoM2D.results;

Mesh = Res.mesh;          % mesh grid
BF   = Res.basisFcns;     % basis functions

% Or without running MoM solver directly from AToM:
Mesh = atom.selectedProject.mesh.getMeshData2D();
BF   = models.solvers.MoM2D.basisFcns.getBasisFcns(Mesh);
```

Impedance matrix(-related) operators $\mathbf{Z}$, $\mathbf{W}$, $\mathbf{Z}^{M,k}$, $\mathbf{Z}^{M,0}$, $\mathbf{Z}^{E,k}$, $\mathbf{Z}^{E,0}$, $\mathbf{T}^{M}$, $\mathbf{T}^{E}$:

Listing 2: atom_code1_MoM1.m

```
% Impedance matrix
Z   = Res.zMat.data;

omega = 2*pi*f;
% Stored energy matrix
W   = omega*imag(Res.zMatD.data); % omega*DZ!

% Individual parts of impedance matrix:
ZMk  = Res.zMatMk.data;
ZM0  = Res.zMatM0.data;
ZEk  = Res.zMatEk.data;
ZE0  = Res.zMatE0.data;
TE   = Res.tMatE.data;
TM   = Res.tMatM.data;
```

Alternatively, **Z** and **W** can be calculated directly from normalized blocks:

Listing 3: atom_code1_MoM2.m

```
% Circumscribing sphere
a  = models.utilities.meshPublic.getCircumsphere(Mesh.nodes);
% Wavenumber and electrical size
k  = models.utilities.converter.f0tok0(f);
ka = k*a;

% Normalization constant
C  = a^2*Z0;

% Impedance matrix and stored energy matrix
Z  = 1j*C*(ka*(ZMk + ZM0) - 1/ka*(ZEk + ZE0));
W  = imag(1j*C*(ka*(ZMk + ZM0 - 1j*ka*TM) + 1/ka*(ZEk + ZE0 + 1j*ka*TE)));
```

Basis-function overlap matrix $\mathbf{\Psi}$ and related matrices $\mathbf{R}_\Omega$ and $\mathbf{A}$:

Listing 4: atom_code1_LossyMatrix.m

```
% The basis-function overlap matrix
Psi = models.utilities.matrixOperators.ohmicLosses.lossyMatrix(Mesh, BF);

% Dissipation losses matrix
Romega = Rs*Psi;

% RWG adjacency matrix
A = double(logical(Psi));
```

Far-field vectors **F** and matrix **U**:

Listing 5: atom_code1_farfield.m

```
% Theta and phi used to specify $\widehat{\boldsymbol{r}}$
theta = pi/2; % from 0 to pi
phi   = 0;    % from 0 to 2*pi
% Component used to specify $\widehat{\boldsymbol{e}}$
component = 'theta'; % component = {'theta' | 'phi' | 'total'}

% Far-field matrix (U) and vectors (F)
[U, F_phi, F_theta] = ...
    models.utilities.matrixOperators.farfield.computeU...
    (Mesh, BF, f, theta, phi, component)
```

Spherical wave projection matrix **S**:

Listing 6: atom_code1_sphericalwaves.m

```
nQuad = 1;  % order of quadrature rule
Lmax  = 15; % highest degree of used Legendre polynomial

% Spherical wave projection matrix of size NxM
% N - number of spherical waves, M - number of RWG functions
S = models.utilities.matrixOperators.SMatrix.computeS...
    (Mesh, BF, f, Lmax, nQuad);
```

Electric and magnetic dipole matrices **P** and **M**, and vectors **p** and **m**:

Listing 7: atom_code1_elmagMoments.m

```
% Electric moment matrix (P) and vector (p)
[P, p] = models.utilities.matrixOperators.electricMoment.computeP...
    (Mesh, BF, f);

% Magnetic moment matrix (M) and vector (m)
[M, m] = models.utilities.matrixOperators.magneticMoment.computeM...
    (Mesh, BF, f);
```

Far-field approximation

$$\boldsymbol{E}_{\mathrm{far}}\left(\boldsymbol{r}\right) = -\mathrm{j}\omega \boldsymbol{A}\left(\boldsymbol{r}\right), \quad \boldsymbol{A}\left(\boldsymbol{r}\right) = \frac{\mu_0}{4\pi} \int\limits_{\Omega} \boldsymbol{J}\left(\boldsymbol{r}'\right) \frac{\mathrm{e}^{-\mathrm{j}k\left|\boldsymbol{r}-\boldsymbol{r}'\right|}}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|} \,\mathrm{d}S, \tag{60}$$

$$\text{with amplitude} \quad R \approx \left|\boldsymbol{r}\right| = r \quad \text{and phase} \quad R \approx r - \hat{\boldsymbol{r}} \cdot \boldsymbol{r}' \tag{61}$$

leads to

$$\boldsymbol{E}_{\mathrm{far}}\left(\boldsymbol{r}\right) = -\frac{\mathrm{j}Z_0 k}{4\pi} \frac{\mathrm{e}^{-\mathrm{j}kr}}{r} \int\limits_{\Omega} \boldsymbol{J}\left(\boldsymbol{r}'\right) \mathrm{e}^{\mathrm{j}k\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'} \,\mathrm{d}S. \tag{62}$$

Questions?

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